Average speed:

Type 1: When speed and distances are given.

Case a: When unequal portion of distances are travelled at varying speeds:

a_1: When each of values of distance portions are given:

Average speed = (Total distance /Total time taken)

When a vehicle travels a certain distance d_1 km at a speed of S_1km/hr and a certain distance d_2 km at a speed of S_2 km/hr, the average speed is given by,

$S = (d_1+d_2)/ ((d_1/S_1) + (d_2/S_2))$….. (1)

Example: When a car travels first 60km at the rate of 40km/hr and the next 80km at the rate of 70km/hr. What is its average speed?

In this case, we will directly apply equation (1).

Hence, average speed,$S = (60+80)/((60/40)+(80/70))=1960/37= 52.973$km/hr.

a_2: When all different distance portions are expressed as a distance ratio of a single portion:

When a vehicle travels a certain distance d_1 km at a speed of S_1 km/hr and another distance $(x*d_1)$ at a speed of S_2 km/hr, the average speed is given by,

$S= (d_1+(x*d_1))/((d_1/S_1)+((x*d_1)/S_2))$

$= ((1+x)(S_1*S_2))/(S_2+(x*S_1))$……(2).

Example: When a car travels a certain distance at 50 km/hr and (1/3) of the initial distance at 90 km/hr, what is its average speed?

Its average speed $S= ((1+(1/3))(50*90))/(90+((1/3)*50))$ (applying equation 2).

= 56.25 km/hr.

Case b: When equal fractions of distances are travelled at varying speeds:

When the vehicle travels (1/2) of the distance at a speed of S_1 km/hr and the remaining (1/2) of the distance at a speed of S_2 km/hr ,

equation (1) becomes $2d_1/((d_1/S_1)+(d_1/S_2))$

$= (2*S_1*S_2)/(S_1+S_2)$……..(3)

Similarly, when the vehicle travels (1/3) of the distance at a speed of S_1 km/hr,second and third (1/3) of the distances at speeds S_2 and S_3 km/hr respectively,

Average speed $= (3*S_1*S_2*S_3)/((S_1*S_2)+(S_2*S_3)+(S_3*S_1))$……..(4)

In general, when a vehicle travels (1/N) of the distance at a speed of S_1 m/hr and the subsequent

(N-1) fractions each of a distance of (1/N) at varying speeds of S_2,S_3,S_4 …..S_N km/hr

respectively,

Average speed= $(N*S_1*S_2*S_3*\ldots\ldots S_N)/((S_1*S_2*S_3*S_4\ldots S_{N-1})+(S_2*S_3*S_4*\ldots..S_N)+$(Sum of similar sequence of(N-2) terms)).

Example: When a vehicle travels first 100 km at a speed of 60 km/hr and the next 100 km at a speed of 80 km/hr,

Its average speed = (2*60*80)/(60+80)= 68.57 km/hr (by applying equation 3).

Type 2: When speed and time are given:

Case a: When ,for unequal time periods, varying speeds are encountered:

a_1: when the value of each of the time periods are specified:

When a vehicle travels at a speed of S_1 km/hr for a time period of t_1 hours and at a speed of S_2 km/hr for a time period of t_2 hours,

Average speed, $S = ((t_1*S_1)+(t_2*S_2))/(t_1+t_2)$……(5).

Example : When a car travels first 4 hours at a speed of 70 km/hr and the next 1.5 hours at a speed of 100 km/hr, what is its average speed?

Its average speed ,S= ((4*70)+(1.5*100))/(4+1.5) (applying equation 5).

= 78.1818 km/hr.

a_2: When all different time periods are expressed as a ratio of single time period:

When a vehicle travels at a speed of S_1 km/hr for a time period of t_1 hours and at a speed of S_2 km/hr for a time period of $(x*t_1)$ hours,

Average speed, $S= ((t_1*S_1)+((x*t_1)*S_2))/(t_1+(x*t_1))$

$=(S_1+(x*S_2))/(1+x)$……(6)

Example: When a car travels at a speed of 40 km/hr for a certain time period and at a speed of 60 km/hr for half the initial time period, what is its average speed?

Its average speed, S= (40+(0.5*60))/1.5 (applying equation 6).

= 46.66 km/hr.

Case b: When, for equal time periods, varying speeds are encountered:

When a vehicle travels for first t_1 hours at a speed of S_1 km/hr and the next t_1 hours at a speed of S_2 km/hr,

Average speed, $S=((t_1*S_1)+(t_1*S_2))/(t_1+t_1)$

$=(S_1+S_2)/2$= Arithmetic average ……..(7)

Example: When a car travels at a speed of 30 km/hr for the first 2 hours and at a speed of 50 km/hr for the next 2 hours, what is its average speed?

Its average speed= (30+50)/2 (applying equation 7)= 40 km/hr.